

PLASMA DYNAMICS

Generation of longitudinal electric current by the transversal electromagnetic field in collisional plasma

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Abstract

Kinetic Vlasov equation for collisional plasmas with BGK (Bhatnagar-Gross-Krook) collision integral is used. The case of arbitrary temperature (i.e. arbitrary degree of degeneration of electronic gas) is considered.

From kinetic Vlasov equation the distribution function in square-law approximation on size of transversal electromagnetic field is received. The formula for calculation electric current is deduced. This formula contains one-dimension quadrature.

It is shown, that the nonlinearity account leads to occurrence the longitudinal electric current directed along the wave vector. This longitudinal current is perpendicular to the known transversal classical current, received at the linear analysis.

When frequency of collisions tends to zero, all received results for collisional plasma pass in known corresponding formulas for collisionless plasma.

The case of small values of wave number is considered. It is shown, that the received quantity of longitudinal current at tendency of frequency of collisions to zero also passes in known corresponding expression of current for collisionless plasmas.

Graphic research of dimensionless density of the current depending on wave number, frequency of oscillations of electromagnetic field and frequency of electron collisions with plasma particles is carried out.

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INTRODUCTION

In the present work formulas for calculation electric current in classical collisional Fermi—Dirac plasma are deduced.

At the solution of the kinetic Vlasov equation we consider as in expansion distribution function, and in expansion of size of the self-consistent electromagnetic field the quantities proportional to square of intensity of external electromagnetic field.

At such nonlinear approach it appears, that an electric current has two nonzero components. One component of electric current is directed along intensity of electric field. This electric current component is precisely the same, as well as in the linear analysis. It is "transversal" current. Thus, in linear approach we receive the known expression of transversal electric current.

The second nonzero electric current component has the second order of infinitesimality concerning intensity of size of electromagnetic field. The second electric current component is directed along the wave vector. This current is orthogonal to the first component. It is "longitudinal" current.

Generating in plasma of the longitudinal current by the transversal electromagnetic field comes to light the nonlinear analysis of interactions of the electromagnetic field with plasma.

Nonlinear effects in plasma already long time [1] – [5] are studied.

In works [1] and [3] nonlinear effects in plasma are studied. In work [3] the nonlinear current was used, in particular, in questions of probability of decay processes. We will note, that in work [2] it is underlined existence of nonlinear current along a wave vector (see the formula (2.9) from [2]).

Quantum plasma was studied in works [6] - [22]. Collisional quantum plasma has started to be studied in work [13]. Then quantum collisional plasma was studied in our works [15] - [19]. In this works quantum collisional plasma with variable collision frequency was studied. In works [20] - [22] generating of longitudinal current by transversal electromagnetic field in classical and quantum Fermi—Dirac plasma [20], in Maxwellian plasma [21] and in degenerate plasma [22] was investigated.

In the present work formulas for calculation electric current are deduced in classical collisionless plasma at any temperature (at the any degrees of degeneration of electronic gas).

1 SOLUTION OF VLASOV EQUATION

Let us show, that in case of the classical plasma described by the Vlasov equation, the longitudinal current is generated and we will calculate its density. On existence of this current was specified more half a century ago [2]. We take Vlasov equation describing behaviour of collisional plasmas with integral of collisions BGK (Bhatnagar, Gross and Krook)

$$\frac{\partial f}{\partial t} + \mathbf{v} \frac{\partial f}{\partial \mathbf{r}} + e \left(\mathbf{E} + \frac{1}{c} [\mathbf{v}, \mathbf{H}] \right) \frac{\partial f}{\partial \mathbf{p}} = \nu (f^{(0)} - f). \quad (1.1)$$

In equation (1.1) $f = f(\mathbf{r}, \mathbf{v}, t)$ is the distribution electron function of plasma, \mathbf{E}, \mathbf{H} are the components of electromagnetic field, c is the light velocity, $\mathbf{p} = m\mathbf{v}$ is the electron momentum, \mathbf{v} is the electron velocity, ν is the effective electron collision frequency with plasma particles, $f^{(0)} = f_{eq}(\mathbf{r}, v)$ (eq \equiv equilibrium) is the local equilibrium Fermi–Dirac distribution,

$$\begin{aligned} f_{eq}(\mathbf{r}, v) &= \left[1 + \exp \frac{\mathcal{E} - \mu(\mathbf{r})}{k_B T} \right]^{-1} = \\ &= \left[1 + \exp(P^2 - \alpha(\mathbf{r})) \right]^{-1} = f_{eq}(\mathbf{r}, P), \end{aligned}$$

$\mathcal{E} = mv^2/2$ is the electron energy, $\mu(\mathbf{r})$ is the chemical potential of electron gas, k_B is the Boltzmann constant, T is the plasma temperature, $\mathbf{P} = \mathbf{p}/p_T$ is the dimensionless electron momentum, $p_T = mv_T$, v_T is the thermal electron velocity,

$$v_T = \sqrt{\frac{2k_B T}{m}}, \quad \alpha(\mathbf{r}) = \frac{\mu(\mathbf{r})}{(k_B T)}$$

is the dimensionless chemical potential,

$$k_B T = \mathcal{E}_T = \frac{mv_T^2}{2}$$

is the thermal kinetic electron energy.

Let us consider, that in plasma there is an electromagnetic field, representing the running harmonious wave

$$\mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{k}\mathbf{r} - \omega t)}, \quad \mathbf{H} = \mathbf{H}_0 e^{i(\mathbf{k}\mathbf{r} - \omega t)}.$$

Let us consider, that the wave vector \mathbf{k} is orthogonal to potential of the electromagnetic field,

$$\mathbf{k}\mathbf{A}(\mathbf{r}, t) = 0.$$

Electric and magnetic fields are connected with the vector potential by equalities

$$\mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{A}}{\partial t}, \quad \mathbf{H} = \text{rot} \mathbf{A}.$$

For definiteness we will consider, that the wave vector is directed along an axis x , and the electric field is directed along an axis y , i.e.

$$\mathbf{k} = k(1, 0, 0), \quad \mathbf{E} = E_y(x, t)(0, 1, 0).$$

Hence,

$$\begin{aligned} \mathbf{E} &= -\frac{1}{c} \frac{\partial \mathbf{A}}{\partial t} = \frac{i\omega}{c} \mathbf{A}, \\ \mathbf{H} &= \frac{ck}{\omega} E_y \cdot (0, 0, 1), \quad [\mathbf{v}, \mathbf{H}] = \frac{ck}{\omega} E_y \cdot (v_y, -v_x, 0), \\ e \left(\mathbf{E} + \frac{1}{c} [\mathbf{v}, \mathbf{H}] \right) \frac{\partial f}{\partial \mathbf{p}} &= \frac{e}{\omega} E_y \left[kv_y \frac{\partial f}{\partial p_x} + (\omega - kv_x) \frac{\partial f}{\partial p_y} \right], \end{aligned}$$

and also

$$[\mathbf{v}, \mathbf{H}] \frac{\partial f_0}{\partial \mathbf{p}} = 0, \quad \text{because} \quad \frac{\partial f_0}{\partial \mathbf{p}} \sim \mathbf{v}.$$

Let us consider linearization of locally equilibrium function of distribution

$$f_{eq}(P, x) = f_0(P) + g(P) \delta \alpha(x),$$

where

$$\begin{aligned} f_0(P) &= [1 + e^{P^2 - \alpha}]^{-1}, \\ \alpha(x) &= \alpha + \delta \alpha(x), \quad \alpha = \text{const}, \\ g(P) &= \frac{\partial f_0(P)}{\partial \alpha} = \frac{e^{P^2 - \alpha}}{(1 + e^{P^2 - \alpha})^2}. \end{aligned}$$

The equation (1.1) can be copied now in the following form

$$\begin{aligned} \frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} + \frac{eE_y}{\omega} \left[kv_y \frac{\partial f}{\partial p_x} + (\omega - kv_x) \frac{\partial f}{\partial p_y} \right] + \nu f = \\ = \nu f_0(P) + g(P) \nu \delta\alpha(x). \end{aligned} \quad (1.2)$$

Size $\delta\alpha(x)$ we will find from the law of preservation of particles number

$$\int (f_{eq} - f) \frac{2d^3p}{(2\pi\hbar)^2} = 0.$$

From this conservation law we receive that

$$\delta\alpha(x) \int g(P) \frac{2d^3p}{(2\pi\hbar)^2} = \int [f - f_0(P)] \frac{2d^3p}{(2\pi\hbar)^2}.$$

From this equation we obtain that

$$\delta\alpha(x) = \frac{\int [f - f_0(P)] d^3P}{\int g(P) d^3P}.$$

We notice that

$$\int g(P) d^3P = 2\pi \int_0^\infty \frac{dP}{1 + e^{P^2 - \alpha}} = \pi \int_{-\infty}^\infty \frac{dP}{1 + e^{P^2 - \alpha}} = \pi \hat{f}_0(\alpha),$$

where

$$\hat{f}_0(\alpha) = \int_{-\infty}^\infty \frac{dP}{1 + e^{P^2 - \alpha}} = 2 \int_0^\infty \frac{dP}{1 + e^{P^2 - \alpha}}.$$

Therefore

$$\delta\alpha(x) = \frac{1}{\pi \hat{f}_0(\alpha)} \int [f - f_0(P)] d^3P.$$

The equation (1.2) will be transformed now to the integrated equation

$$\begin{aligned} \frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} + \nu f = \nu f_0(P) - \frac{eE_y}{\omega} \left[kv_y \frac{\partial f}{\partial p_x} + (\omega - kv_x) \frac{\partial f}{\partial p_y} \right] + \\ + \nu g(P) \frac{1}{\pi \hat{f}_0(\alpha)} \int [f - f_0(P)] d^3P. \end{aligned} \quad (1.3)$$

Let us search for the solution of the equation (1.3) in the form

$$f = f_0(P) + f_1 + f_2, \quad (1.4)$$

where

$$\begin{aligned} f_1 &\sim E_y \sim e^{i(kx - \omega t)}, \\ f_2 &\sim E_y^2 \sim e^{2i(kx - \omega t)}. \end{aligned}$$

Let us operate with method consecutive approximations, considering as small parameter size of intensity of electric field. Then the equation (1.3) with help (1.4) equivalent to the following equations

$$\begin{aligned} &\frac{\partial f_1}{\partial t} + v_x \frac{\partial f_1}{\partial x} + \nu f_1 = \\ &= -\frac{eE_y}{\omega} \left[kv_y \frac{\partial f_0}{\partial p_x} + (\omega - kv_x) \frac{\partial f_0}{\partial p_y} \right] + \nu g(P) \frac{1}{\pi \hat{f}_0(\alpha)} \int f_1 d^3 P. \end{aligned} \quad (1.5)$$

and

$$\begin{aligned} &\frac{\partial f_2}{\partial t} + v_x \frac{\partial f_2}{\partial x} + \nu f_2 = \\ &= -\frac{eE_y}{\omega} \left[kv_y \frac{\partial f_1}{\partial p_x} + (\omega - kv_x) \frac{\partial f_1}{\partial p_y} \right] + \nu g(P) \frac{1}{\pi \hat{f}_0(\alpha)} \int f_2 d^3 P. \end{aligned} \quad (1.6)$$

From equation (1.5) we obtain that

$$\begin{aligned} &(\nu - i\omega + ikv_x) f_1 = \\ &= -\frac{eE_y}{\omega} \left[kv_y \frac{\partial f_0}{\partial p_x} + (\omega - kv_x) \frac{\partial f_0}{\partial p_y} \right] + \nu g(P) A_1. \end{aligned}$$

Here

$$A_1 = \frac{1}{\pi \hat{f}_0(\alpha)} \int f_1 d^3 P. \quad (1.7)$$

Let us enter dimensionless parametres

$$\Omega = \frac{\omega}{k_T v_T}, \quad y = \frac{\nu}{k_T v_T}, \quad q = \frac{k}{k_T}.$$

Here q is the dimensionless wave number, $k_T = \frac{mv_T}{\hbar}$ is the thermal wave number, Ω is the dimensionless frequency of oscillations of the electromagnetic field.

In the previous equation we will pass to dimensionless parameters

$$\begin{aligned} i(qP_x - z)f_1 = \\ = -\frac{eE_y}{\Omega k_T p_T v_T} \left[qP_y \frac{\partial f_0}{\partial P_x} + (\Omega - qP_x) \frac{\partial f_0}{\partial P_y} \right] + yg(P)A_1. \end{aligned} \quad (1.8)$$

Here

$$z = \Omega + iy = \frac{\omega + iy}{k_T v_T}.$$

We notice that

$$\frac{\partial f_0}{\partial P_x} \sim P_x, \quad \frac{\partial f_0}{\partial P_y} \sim P_y.$$

Hence

$$\left[qP_y \frac{\partial f_0}{\partial P_x} + (\Omega - qP_x) \frac{\partial f_0}{\partial P_y} \right] = \Omega \frac{\partial f_0}{\partial P_y}.$$

Now from the equation (1.8) we find, that

$$f_1 = \frac{ieE_y}{k_T p_T v_T} \cdot \frac{\partial f_0 / \partial P_y}{qP_x - z} - iy \cdot \frac{g(P)}{qP_x - z} A_1. \quad (1.9)$$

Let us substitute (1.9) in the equation (1.7). We receive equality

$$A_1 \left(1 + \frac{iy}{2\pi \hat{f}_0(\alpha)} \int \frac{g(P) d^3 P}{qP_x - z} \right) = \frac{ieE_y}{k_T p_T v_T} \int \frac{\partial f_0 / \partial P_y}{qP_x - z} d^3 P.$$

It is easy to see, that integral in the right part of this equality is equal to zero. Hence

$$A_1 = 0.$$

Thus, it agree (1.9) function f_1 is constructed and defined by equality

$$f_1 = \frac{ieE_y}{k_T p_T v_T} \cdot \frac{\partial f_0 / \partial P_y}{qP_x - z}. \quad (1.10)$$

In the second approximation we substitute f_1 according (1.10) in the equation (1.6).

We will receive the equation

$$\begin{aligned}
& (\nu - 2i\omega + 2ikv_x)f_2 = \\
& -\frac{ie^2E_y^2}{k_T p_T v_T \omega} \left[kv_y \frac{\partial}{\partial p_x} \left(\frac{\partial f_0 / \partial P_y}{qP_x - z} \right) + (\omega - kv_x) \frac{\partial}{\partial p_y} \left(\frac{\partial f_0 / \partial P_y}{qP_x - z} \right) \right] + \\
& + \nu g(P) A_2.
\end{aligned}$$

Here

$$A_2 = \frac{1}{\pi \hat{f}_0(\alpha)} \int f_2 d^3 P. \quad (1.11)$$

Let us pass in this equation to dimensionless parameters. We receive the equation

$$\begin{aligned}
& 2i(qP_x - x - \frac{iy}{2})f_2 = \\
& = -\frac{ie^2E_y^2}{\Omega k_T^2 p_T^2 v_T^2} \left[qP_x \frac{\partial}{\partial P_x} \left(\frac{\partial f_0 / \partial P_y}{qP_x - z} \right) + (\Omega - qP_x) \frac{\partial}{\partial P_y} \left(\frac{\partial f_0 / \partial P_y}{qP_x - z} \right) \right] + \\
& + yg(P) A_2.
\end{aligned}$$

Let us designate

$$z' = \Omega + \frac{iy}{2} = \frac{\omega}{k_T v_T} + i \frac{\nu}{2k_T v_T} = \frac{\omega + i\nu/2}{k_T v_T}.$$

From last equation it is found

$$\begin{aligned}
f_2 = & -\frac{e^2 E_y^2}{2k_T^2 p_T^2 v_T^2 \Omega} \left[qP_y \frac{\partial}{\partial P_x} \left(\frac{\partial f_0 / \partial P_y}{qP_x - z} \right) + \frac{\Omega - qP_x}{qP_x - z} \frac{\partial^2 f_0}{\partial P_y^2} \right] \frac{1}{qP_x - z'} - \\
& - \frac{iy}{2} \cdot \frac{g(P)}{qP_x - z'} A_2.
\end{aligned} \quad (1.12)$$

For finding A_2 we will substitute (1.12) in (1.11). From the received relation it is found A_2

$$A_2 = -\frac{e^2 E_y^2}{2k_T^2 p_T^2 v_T^2 \Omega} \cdot \frac{J_1}{\pi \hat{f}_0(\alpha) + \frac{iy}{2} J_0}.$$

Here

$$J_0 = \int \frac{g(P) d^3 P}{qP_x - z'},$$

$$J_1 = \int \left[qP_y \frac{\partial}{\partial P_x} \left(\frac{\partial f_0 / \partial P_y}{qP_x - z} \right) + \frac{\Omega - qP_x}{qP_x - z} \frac{\partial^2 f_0}{\partial P_y^2} \right] \frac{d^3 P}{qP_x - z'}$$

Substituting the found value A_2 in (1.12), definitively we find function f_2 in the explicit form

$$f_2 = -\frac{e^2 E_y^2}{2k_T^2 p_T^2 v_T^2 \Omega} \left[qP_y \frac{\partial}{\partial P_x} \left(\frac{\partial f_0 / \partial P_y}{qP_x - z} \right) + \frac{\Omega - qP_x}{qP_x - z} \frac{\partial^2 f_0}{\partial P_y^2} \right] \frac{1}{qP_x - z'} +$$

$$+ \gamma \frac{e^2 E_y^2}{2k_T^2 p_T^2 v_T^2 \Omega} \cdot \frac{g(P)}{qP_x - z'}, \quad (1.13)$$

where

$$\gamma = \frac{(iy/2)J_1}{\pi \hat{f}_0(\alpha) + (iy/2)J_0}. \quad (1.14)$$

2 DENSITY OF ELECTRIC CURRENT

Let us find electric current density

$$\mathbf{j} = e \int \mathbf{v} f \frac{2d^3 p}{(2\pi\hbar)^3}. \quad (2.1)$$

From equalities (1.4) – (1.6) it is visible, that the vector of current density has two nonzero components

$$\mathbf{j} = (j_x, j_y, 0).$$

Here j_y is the density of transversal current,

$$j_y = e \int v_y f \frac{2d^3 p}{(2\pi\hbar)^3} = e \int v_y f_1 \frac{2d^3 p}{(2\pi\hbar)^3}.$$

This current is directed along an electric field, its density it is defined only by the first approximation of distribution function.

The second approximation of distribution function the contribution to current density does not bring.

The density of transversal current is defined by equality

$$j_y = \frac{2ie^2 p_T^2}{(2\pi\hbar)^3 k_T} E_y(x, t) \int \frac{(\partial f_0 / \partial P_y) P_y}{qP_x - z} d^3 P.$$

This current is proportional to the first degree of size of electric field intensity.

For density of longitudinal current according to its definition it is had

$$j_x = e \int v_x f \frac{2d^3p}{(2\pi\hbar)^3} = e \int v_x f_2 \frac{2d^3p}{(2\pi\hbar)^3} = \frac{2ev_T p_T^3}{(2\pi\hbar)^3} \int P_x f_2 d^3P.$$

By means of (1.6) from here it is received, that

$$j_x = \frac{e^3 E_y^2 m}{(2\pi\hbar)^3 k_T^2 \Omega} \left[- \int \left[q P_y \frac{\partial}{\partial P_x} \left(\frac{\partial f_0 / \partial P_y}{q P_x - z} \right) + \frac{x - q P_x}{q P_x - z} \frac{\partial^2 f_0}{\partial P_y^2} \right] \frac{P_x d^3 P}{q P_x - z'} + \gamma \int \frac{P_x g(P) d^3 P}{q P_x - z'} \right]. \quad (2.2)$$

In integral from the second composed from square brackets (2.2) internal integral on P_y it is equal to zero:

$$\int_{-\infty}^{\infty} \frac{\partial^2 f_0}{\partial P_y^2} dP_y = \left. \frac{\partial f_0}{\partial P_y} \right|_{P_y=-\infty}^{P_y=+\infty} = 0.$$

In the first integral from square brackets (2.2) internal integral on P_x is calculated in parts

$$\int_{-\infty}^{\infty} \frac{\partial}{\partial P_x} \left(\frac{\partial f_0 / \partial P_y}{q P_x - z} \right) \frac{P_x dP_x}{q P_x - z'} = z' \int_{-\infty}^{\infty} \frac{\partial f_0 / \partial P_y}{(q P_x - z')^2 (q P_x - z)} dP_x.$$

Hence, equality (2.2) becomes simpler

$$j_x = \frac{e^3 E_y^2 m}{(2\pi\hbar)^3 k_T^2 \Omega} \left[- z' q \int \frac{P_y (\partial f_0 / \partial P_y) d^3 P}{(q P_x - z')^2 (q P_x - z)} + \gamma \int \frac{P_x g(P) d^3 P}{q P_x - z'} \right].$$

Internal integral on variable P_y we will integrate on parts

$$\int_{-\infty}^{\infty} P_y \frac{\partial f_0}{\partial P_y} dP_y = P_y f_0 \Big|_{P_y=-\infty}^{P_y=+\infty} - \int_{-\infty}^{\infty} f_0(P) dP_y = - \int_{-\infty}^{\infty} f_0(P) dP_y.$$

Thus, expression for the longitudinal current becomes

$$j_x = \frac{e^3 E_y^2 m}{(2\pi\hbar)^3 k_T^2 \Omega} \left[z' q \int \frac{f_0(P) d^3 P}{(qP_x - z')^2 (qP_x - z)} + \gamma \int \frac{P_x g(P) d^3 P}{qP_x - z'} \right]. \quad (2.3)$$

Internal integral in plane (P_y, P_z) we will calculate in the polar coordinates

$$\begin{aligned} \int \frac{f_0(P) d^3 P}{(qP_x - z')^2 (qP_x - z)} &= \int_{-\infty}^{\infty} \frac{dP_x}{(qP_x - z')^2 (qP_x - z)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_0(P) dP_y dP_z = \\ &= \pi \int_{-\infty}^{\infty} \frac{\ln(1 + e^{\alpha - P_x^2}) dP_x}{(qP_x - z')^2 (qP_x - z)}, \end{aligned}$$

since

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_0(P) dP_y dP_z = \pi \ln(1 + e^{\alpha - P_x^2}).$$

In addition

$$\begin{aligned} \int \frac{P_x g(P) d^3 P}{qP_x - z'} &= \pi \int_{-\infty}^{\infty} \frac{P_x f_0(P_x) dP_x}{qP_x - z'} = \\ &= \pi \int_{-\infty}^{\infty} \frac{\tau d\tau}{(1 + e^{\tau^2 - \alpha})(q\tau - z')} = \pi \int_{-\infty}^{\infty} \frac{e^{\alpha - \tau^2} \tau d\tau}{(1 + e^{\alpha - \tau^2})(q\tau - z')} = \\ &= -\frac{\pi q}{2} \int_{-\infty}^{\infty} \frac{\ln(1 + e^{\alpha - \tau^2}) d\tau}{(q\tau - z')^2}. \end{aligned}$$

Equality (2.3) is reduced to one-dimensional integral

$$j_x = \frac{\pi e^3 E_y^2 m q}{(2\pi\hbar)^3 k_T^2 \Omega} \left[z' \int_{-\infty}^{\infty} \frac{\ln(1 + e^{\alpha - \tau^2}) d\tau}{(q\tau - z')^2 (q\tau - z)} - \frac{\gamma}{2} \int_{-\infty}^{\infty} \frac{\ln(1 + e^{\alpha - \tau^2}) d\tau}{(q\tau - z')^2} \right].$$

Let us rewrite the previous equality in the form

$$j_x = \frac{\pi e^3 E_y^2 m q}{(2\pi\hbar)^3 k_T^2 \Omega} \left[z' J_{12} - \frac{\gamma}{2} J_{02} \right]. \quad (2.4)$$

where

$$J_{12} = \int_{-\infty}^{\infty} \frac{\ln(1 + e^{\alpha - \tau^2}) d\tau}{(qP_x - z)(q\tau - z')^2},$$

and

$$J_{02} = \int_{-\infty}^{\infty} \frac{\ln(1 + e^{\alpha - \tau^2}) d\tau}{(q\tau - z')^2}.$$

Let us return to consideration of size γ . We will calculate integrals, entering in (1.14). We will calculate the first integral

$$J_1 = \int \left[qP_y \frac{\partial}{\partial P_x} \left(\frac{\partial f_0 / \partial P_y}{qP_x - z} \right) + \frac{x - qP_x}{qP_x - z} \frac{\partial^2 f_0}{\partial P_y^2} \right] \frac{d^3 P}{qP_x - z'}.$$

As it was already specified, the integral from the second composed is equal to zero. The second integral as well as earlier we will calculate in parts. As a result we receive

$$\begin{aligned} J_1 &= q \int P_y \frac{\partial}{\partial P_x} \left(\frac{\partial f_0 / \partial P_y}{qP_x - z} \right) \frac{d^3 P}{qP_x - z'} = \\ &= q^2 \int \frac{P_y [\partial f_0 / \partial P_y] d^3 P}{(qP_x - z)(qP_x - z')^2}. \end{aligned}$$

Now we will calculate in parts internal integral on the variable P_y . As the result we receive

$$J_1 = -q^2 \int \frac{f_0(P) d^3 P}{(qP_x - z)(qP_x - z')^2}.$$

This integral has been calculated earlier. Hence

$$J_1 = -\pi q^2 \int_{-\infty}^{\infty} \frac{\ln(1 + e^{\alpha - \tau^2}) d\tau}{(qP_x - z)(q\tau - z')^2} = -\pi q^2 J_{12}.$$

Let us calculate the second integral from (1.14). We have

$$J_0 = \int \frac{g(P)d^3P}{qP_x - z'} = \int_{-\infty}^{\infty} \frac{dP_x}{qP_x - z'} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(P)dP_y dP_z.$$

The internal double integral is equal

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(P)dP_y dP_z = \pi \frac{1}{1 + e^{P^2 - \alpha}} = \pi f_0(P).$$

Hence, the integral J_0 is equal

$$J_0 = \pi \int_{-\infty}^{\infty} \frac{f_0(\tau)d\tau}{q\tau - z'}.$$

So

$$\pi \hat{f}_0(\alpha) + \frac{iy}{2} J_0 = \pi \int_{-\infty}^{\infty} \frac{q\tau - \Omega}{q\tau - z'} f_0(\tau) d\tau.$$

Thus, the constant γ is found

$$\gamma = -\frac{iy}{2} q^2 \frac{J_{12}}{J_{01}},$$

where

$$J_{01} = \int_{-\infty}^{\infty} \frac{q\tau - \Omega}{q\tau - z'} f_0(\tau) d\tau.$$

Now (2.4) it is possible to present the formula in the form

$$j_x = \frac{\pi e^3 E_y^2 m q}{(2\pi\hbar)^3 k_T^2 \Omega} \left[\Omega + \frac{iy}{2} + \frac{iy}{4} q^2 \frac{J_{02}}{J_{01}} \right] J_{12}. \quad (2.4')$$

Let us find numerical density (concentration) of plasma particles, corresponding to Fermi—Dirac distribution

$$N = \int f_0(P) \frac{2d^3p}{(2\pi\hbar)^3} = \frac{8\pi p_T^3}{(2\pi\hbar)^3} \int_0^{\infty} \frac{e^{\alpha - P^2} P^2 dP}{1 + e^{\alpha - P^2}} = \frac{k_T^3}{2\pi^2} l_0(\alpha),$$

where k_T is the thermal wave number, $k_T = \frac{mv_T}{\hbar}$,

$$l_0(\alpha) = \int_0^\infty \ln(1 + e^{\alpha - \tau^2}) d\tau.$$

In expression before integral from (2.4) we will allocate the plasma (Langmuir) frequency

$$\omega_p = \sqrt{\frac{4\pi e^2 N}{m}}$$

and number density (concentration) N , and last we will express through thermal wave number. We will receive

$$j_x^{\text{long}} = \left(\frac{e\Omega_p^2}{k_T p_T} \right) \frac{k E_y^2}{16\pi l_0(\alpha) \Omega} \left[\Omega + \frac{iy}{2} + \frac{iy}{4} q^2 \frac{J_{02}}{J_{01}} \right] J_{12}, \quad (2.5)$$

where

$$\Omega_p = \frac{\omega_p}{k_T v_T} = \frac{\hbar \omega_p}{m v_T^2}$$

is the dimensionless plasma frequency.

Equality (2.5) we rewrite in the form

$$j_x^{\text{long}} = J(\Omega, y, q) \sigma_{l, tr} k E_y^2, \quad (2.6)$$

where $\sigma_{l, tr}$ is the longitudinal-transversal conductivity, $J(\Omega, y, q)$ is the dimensionless part of current,

$$\sigma_{l, tr} = \frac{e\Omega_p^2}{p_T k_T} = \frac{e\hbar}{p_T^2} \left(\frac{\hbar \omega_p}{m v_T^2} \right)^2 = \frac{e}{k_T p_T} \left(\frac{\omega_p}{k_T v_T} \right)^2,$$

$$J(\Omega, y, q) = \frac{1}{16\pi l_0(\alpha) \Omega} \left[\Omega + \frac{iy}{2} + \frac{iy}{4} q^2 \frac{J_{02}}{J_{01}} \right] J_{12}.$$

If to enter transversal field

$$\mathbf{E}_{tr} = \mathbf{E} - \frac{\mathbf{k}(\mathbf{E}\mathbf{k})}{k^2} = \mathbf{E} - \frac{\mathbf{q}(\mathbf{E}\mathbf{q})}{q^2}, \quad \mathbf{k}\mathbf{E}_{tr} = \frac{\omega}{c} [\mathbf{E}, \mathbf{H}],$$

then the equality (2.6) we will to write down in invariant form

$$\mathbf{j}^{\text{long}} = J(\Omega, y, q) \sigma_{l, tr} \mathbf{k}\mathbf{E}_{tr}^2 = J(\Omega, y, q) \sigma_{l, tr} \frac{\omega}{c} [\mathbf{E}, \mathbf{H}].$$

REMARK. From the formula (2.5) (or from (2.6)) it is visible, that at $y = 0$ (or $\nu = 0$) i.e. when collisional plasma passes in collisionless ($z \rightarrow \Omega, z' \rightarrow \Omega$), this formula in accuracy passes in the corresponding formula from our work [20] for collisionless plasmas

$$j_x^{\text{long}} = \sigma_{1,\text{tr}} k E_y^2 \frac{1}{16\pi l_0(\alpha)} \int_{-\infty}^{\infty} \frac{\ln(1 + e^{\alpha - \tau^2}) d\tau}{(q\tau - \Omega)^2 (q\tau - \Omega)},$$

Let us pass to consideration of the case of small values of wave number. From expression (2.5) at small values of wave number it is received

$$\begin{aligned} j_x^{\text{long}} &= -\sigma_{1,\text{tr}} k E_y^2 \frac{1}{16\pi l_0(\alpha) \Omega z z'} \int_{-\infty}^{\infty} \ln(1 + e^{\alpha - \tau^2}) d\tau = \\ &= -\frac{\sigma_{1,\text{tr}} k E_y^2}{8\pi \Omega z z'} = -\frac{e}{8\pi m \omega} \left(\frac{\omega_p}{\omega} \right)^2 \frac{k E_y^2}{\left(1 - i \frac{\nu}{\omega}\right) \left(1 - i \frac{\nu}{2\omega}\right)}. \end{aligned}$$

REMARK. At $\nu = 0$ from this formula in accuracy turns out the corresponding formula from [20] for longitudinal current in the case of small values of wave number in collisionless plasma.

3 CONCLUSION

On Figs. 1 and 2 we will present behaviour of real (Fig. 1) and imaginary (Fig. 2) parts of density of the dimensionless longitudinal current at $\Omega = 1, y = 0.01$ depending on dimensionless wave number q at various values of dimensionless chemical potential. At small and great values of parametre q curves 1,2 and 3 approach and become indiscernible. The real part has at first the minimum, and then the maximum. With growth of the dimensionless chemical potential the imaginary part of density of a current has one maximum.

Further graphic research of size of density of the longitudinal current let us spend for the case of zero chemical potential: $\alpha = 0$ (Figs. 3 – 6).

On Figs. 3 and 4 we will present behaviour of real (Fig. 3) and imaginary (Fig. 4) parts of density of the longitudinal current depending on dimensionless wave numbers q in the case $\Omega = 1$ at various values of dimensionless frequency of electron collisions. At small and at great values of dimensionless wave number curves 1,2 and 3 approach and become indiscernible.

On Figs. 5 and 6 we will present behaviour of real (Fig. 5) and imaginary (Fig. 6) parts of density of the longitudinal current in dependence from dimensionless frequency of oscillations of the electromagnetic field Ω in the case $q = 0.3$. At increase of dimensionless wave number q curves 1,2 and 3 approach and at $q > 0.7$ practically coincide.

In the present work influence of nonlinear character of interactions of the electromagnetic field with the classical collisional plasma is considered.

It has appeared, that presence of nonlinearity of the electromagnetic field leads to generating of the electric current, orthogonal to the field direction.

Further authors purpose to consider a problem of the plasma oscillations and a problem about skin-effect with use square vector potential in expansion of distribution function.

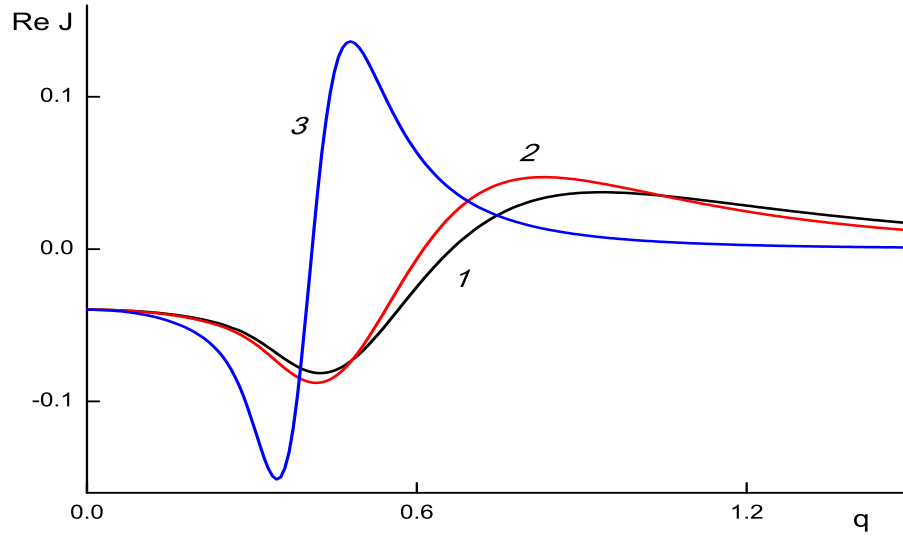


Fig. 1. Real part of dimensionless density of longitudinal current, $\Omega = 1, y = 0.01$. Curves 1, 2, 3 correspond to values of dimensionless chemical potential $\alpha = -5, 0, +5$.

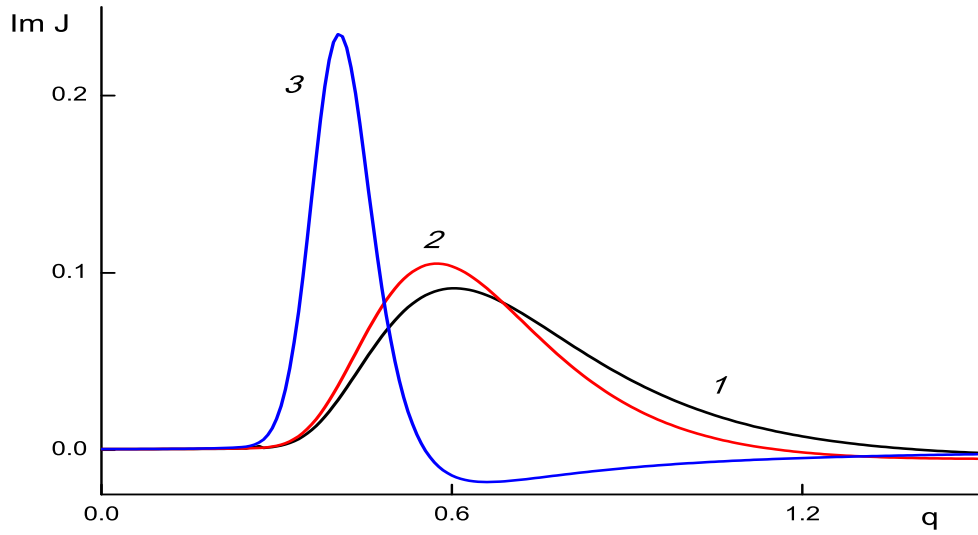


Fig. 2. Imaginary part of dimensionless density of longitudinal current, $\Omega = 1, y = 0.01$. Curves 1, 2, 3 correspond to values of dimensionless chemical potential $\alpha = -5, 0, +5$.

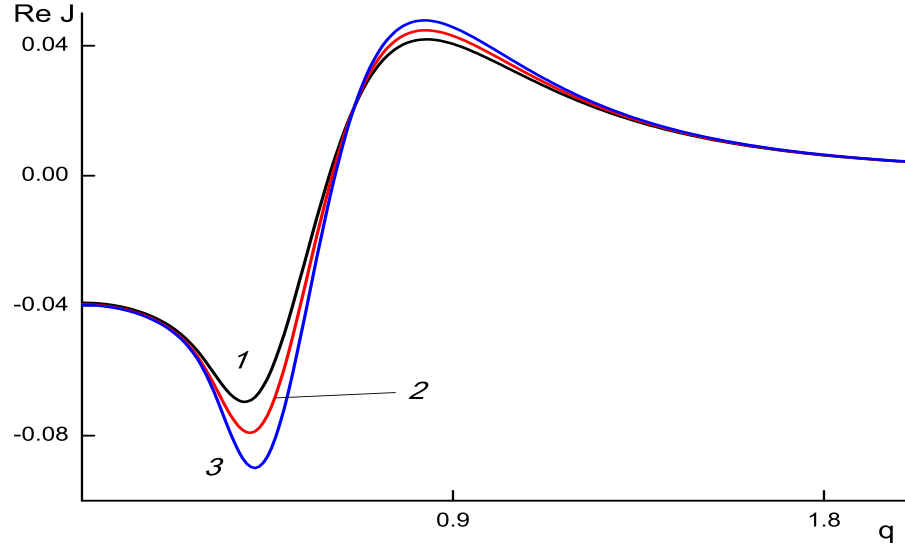


Fig. 3. Real part of dimensionless density of longitudinal current, $\Omega = 1, \alpha = 0$.
Curves 1, 2, 3 correspond to values of dimensionless collision frequency
 $y = 0.001, 0.05, 0.1$.

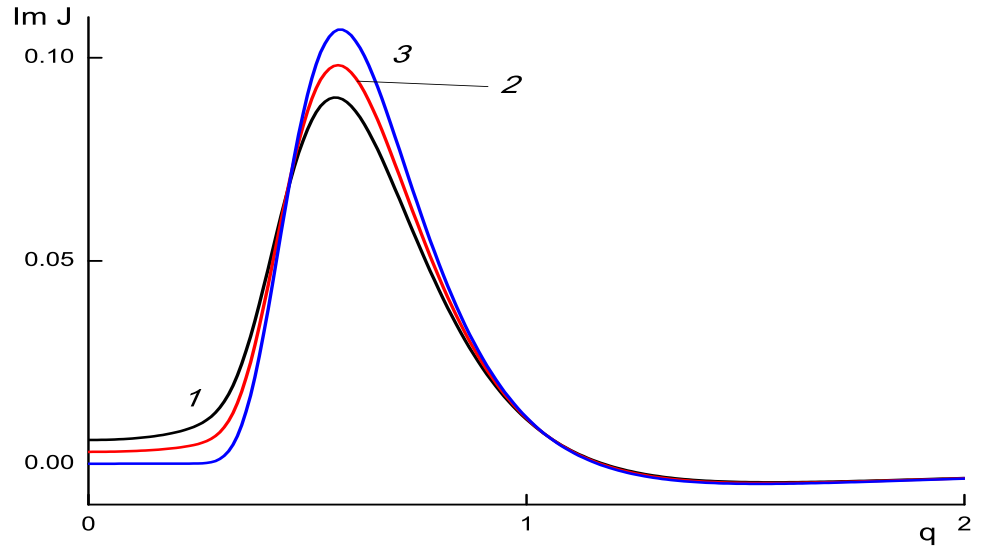


Fig. 4. Imaginary part of dimensionless density of longitudinal current, $\Omega = 1, \alpha = 0$.
Curves 1, 2, 3 correspond to values of dimensionless collision frequency
 $y = 0.001, 0.05, 0.1$.

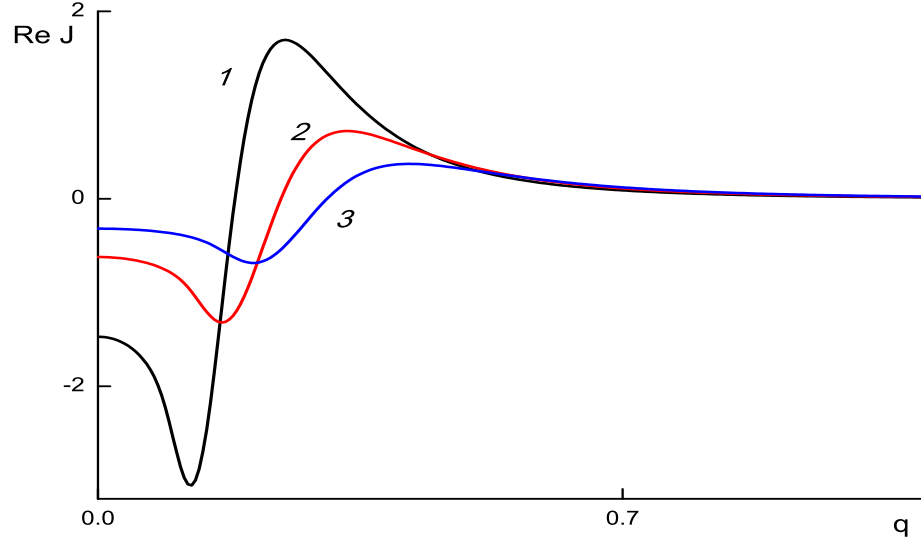


Fig. 5. Real part of dimensionless density of longitudinal current, $y = 0.01, \alpha = 0$.
Curves 1, 2, 3 correspond to values of dimensionless frequency of oscillations
of electromagnetic field $\Omega = 0.3, 0.4, 0.5$.

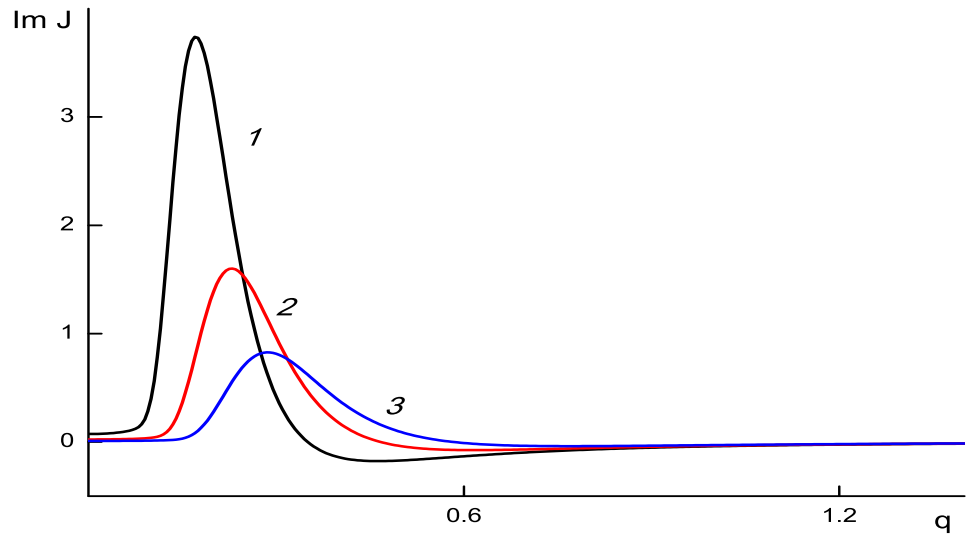


Fig. 6. Imaginary part of dimensionless density of longitudinal current, $x = 1, \alpha = 0$.
Curves 1, 2, 3 correspond to values of dimensionless frequency of oscillations of
electromagnetic field $\Omega = 0.3, 0.4, 0.5$.

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